SOAR Research Proposal – Summer 2017
Parameter Dependent Network Reliability Measures

Faculty: Nathan Shank, Associate Professor, Mathematics and Computer Science Department
Dates: May 30, 2017 - August 4, 2017 (10 weeks)
Student: Makkah Davis, Junior Mathematics Major - Secondary Education
Graduation: Spring 2018

Title: Parameter Dependent Network Reliability Measures

Description of the Project

Network reliability has become an increasingly popular multidisciplinary problem. Computer scientists need to know how to make a computer network secure, health professionals need to understand how social network structures influence the spread of disease, and business administration professionals need to understand how employee network can be constructed to maximize profit and/or efficiency. In this project we will consider how different network properties influence the reliability of networks based on their structure.

Mathematicians have been studying theoretical measures of network reliability for about 50 years. In many applications a network can remain operational as long as some property P is satisfied. For example, if we are considering GPS tracking, as long as three satellites can locate a device, then the position of the device can be retrieved. In another application a network may be operational as long as there are k batteries are connected and thus you have enough power for your system to function. If a network is not operational, i.e. property P is not present in the network, then we say a network is in a failure state. In each of these problems, it is useful to understand how easily a network can be put into a failure state.

At its simplest form, a network is a collection of vertices and edges connecting some of the vertices. The vertices may represent computers, people, countries, power supplies, etc.. In some applications the vertices fail (like a computer on a campus network) and in other applications the edges fail (like a power lines connecting power stations). In some instances edges and vertices may fail (like telephone lines and telephone poles). Although these applications are very useful for motivation, mathematicians try to simplify problems to an abstract structure so we can apply results in many different ways. This is why mathematicians view networks as a set of vertices, V, and a set of edges E.

There are two things to consider when we talk about network reliability measures. First, what is the condition that makes a network operational? Again, assume our network is operational as long as property P is satisfied, then a network is in a failure state if property P is not satisfied. So the property P plays a crucial role in measuring network reliability. Second, what part of the network is prone to failure? Are the edges, vertices, or both able to fail? Mathematicians are interested in several questions related to these two concepts including:

1. If we can only afford n vertices and m edges, what is the least and most reliable network we can construct?

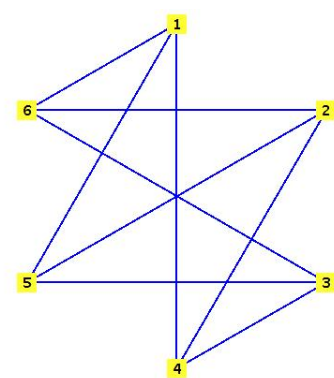

Figure 1: *An example graph*

2. Given a network structure, can we find the least number of failure parts (edges or vertices) needed to render the network into a failure state?
3. If parts of our network randomly fail over time, what is the expected life of a network?

Although all of these applications are trying to put a network into a failure state, there are some applications which are very important for keeping things alive. For example, consider a social network of people. Assume a virus can spread throughout the network and the virus becomes deadly if it passes through k people. Our goal is to stop the spread of the disease from becoming deadly. So we have two options; to quarantine which is equivalent to deleting edges, or to vaccinate which is equivalent to vertex deletions. Finding if edge failures or vertex failures renders the network into a failure state quickest is an important question in epidemiology. Two years ago, my SOAR and Honors student, Adam Buzzard, considered this exact question. This turned out to be the first known example where edge deletions can be faster than vertex deletions. This was an unanswered question for almost 20 years. The results of Adam's work are contained in two papers. The first paper has been submitted for review to a mathematics journal.

During this SOAR project, Makkah will continue some of the work that Adam completed. She will be looking at different properties, P, and trying to find the network reliability parameters based on different failure models, including the mixed model where edges and vertices can fail.

The main goal of this project is to have Makkah understand and experience what it means to do mathematical research. This is especially important to her career in secondary education. I was inspired by my high school mathematics teacher and I want Makkah to be able to give her students the same inspiration and encouragement when they become curious about unsolved mathematics problems. Too often K-12 students think mathematics is just routine calculations. Makkah's experience in SOAR will allow her to better engage students in mathematics and give them an understanding of why mathematics is important in many different disciplines.

If funded, Makkah will also participate in the Muhlenberg Mathematics REU program. The program was designed so that if a LVAIC mathematics student received funding for summer research from their home institution, then they are eligible to participate in the REU program.

The REU program consists of approximately 25 gifted mathematics students from all over the county. The students work in small groups on a mathematical project with two faculty mentors. Dr. Talbott and I are two of the five faculty mentors for the program this summer. So in addition to doing the project outlined above, Makkah will be able to choose another project based on the proposed projects of the REU program.

One particular proposed project which interests both Makkah (and myself) is on Quasi Factor Pair Latin Squares. Assume we are given that $n = 12$. Can we find a Sudoku solution on a 12 grid so that every factor pair of 12 satisfies the criteria of a Sudoku puzzle? This means that subsquares of size 3×4 (since $3 \cdot 4 = 12$) must form a Sudoku puzzle, as well as subsquares of 1×12 (since $1 \cdot 12 = 12$) and subsquares of 2×6 (since $2 \cdot 6 = 12$). The question of interest is what values of n have a Quasi Factor Pair Latin Square solution.

Roles and Responsibilities

Makkah and I will share responsibilities on all aspects of the project, although we may devote varying amounts of time to each part. Her literature review will begin as soon as the spring semester is completed. She will be looking into different reliability measures, reading Adam's papers, and learning some of the proof techniques. She will be responsible for finding papers and summarizing them for our first meeting. This will serve as an introduction to the mathematical concepts used in analyzing network reliability.

After the literature review, we will spend time working on different properties P and seeing how the property works on basic graph classes. This will involve some theoretical work, but also lots of example graphs. We will then develop some conjectures based on different properties P and then work to prove the conjectures. Here is where we may have to adjust our parameter based on how difficult the problems become. This is an important step for a student to understand when conducting mathematical research. As mathematicians, we set out to solve a problem, but usually underestimate the difficulty of the program. Therefore we have to adjust or simplify our original problem until it is something we can solve. Then we gradually make the problem more difficult.

1. Literature Review (Both):
2. Develop Conjectures (Both):
3. Prove Conjectures (Both):
4. Written Results (Both):

Timetable

- 1 weeks: Background reading
- 2 weeks: Developing conjectures based on readings and current work
- 5 weeks: Attempt to prove conjectures and making appropriate adjustments
- 2 weeks: Write up results and prepare presentations

Engagement in Discipline-Appropriate Scholarly Research

This project will follow the general outline of pure mathematical research. This includes gaining background information, developing sound definitions and rules, analyzing or proving results based on these definitions and rules, and communicating (both written and verbally) results to different audiences. Makkah will have a major role in all phases of the project.

Contribution to the Discipline and Opportunities to Share Work

This project will contribute knowledge to the general understanding of network reliability measures which engages many different disciplines. I expect at least one peer reviewed journal paper and several presentations as a result of this SOAR / REU project. Makkah will share his work at the regional *Eastern Pennsylvania and Delaware* section meeting of the *Mathematical Association of America*, Moravian Scholars Day, and the annual *Moravian Student Mathematics Conference* in February.

Summary of Benefits

The benefits of this project for the student include developing the students understanding of mathematical research, strengthening the students credentials for being an effective high school teacher, and improving upon the level of undergraduate education for the students. For me, the benefits include developing skills in mentoring undergraduates, contributing significant results to the general mathematics field, and engaging students in several areas of my research at the same time. For the department and the college the benefits include increasing mathematical awareness on campus and enhancing the scholarly atmosphere.

SOAR Student Statement – Summer 2017
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Student: Makkah Davis, Mathematics Major
Graduation: Spring 2018
Housing: NO

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Rationale

My parents have always encouraged me to learn through exploration and experience, while never failing to be a source of support which enables me to be secure in my abilities (including its potential as well as boundaries). These values have lead to a true belief in embracing challenges as a means of growth, however, also being unafraid to admit that challenge is bound to entail difficulty and require assistance. The SOAR Summer Research program and the REU program present themselves as amazing opportunities for me to continue to actualize such values. Both programs would afford me the opportunity to explore, discover, and expand my knowledge in various mathematical concepts, which is essential to me both a student and an aspiring teacher. A vital benefit of working on this project, for me, is the exposure it will grant me, both in terms of mathematical concepts and with respect to collaborating with other mathematical minds. Working alongside peers and faculty in a collegial manner allows the opportunity to be engaged in meaningful research with people on whom I can rely to support and bolster my academic growth.

The symbiotic nature of teaching and learning through collaboration is an indispensable opportunity that the SOAR project offers. Collaborating with fellow peers and faculty, both within and outside of my discipline, will promote a love and understanding for all aspects of academic research. SOAR will afford me the opportunity to participate in mathematical research and explore relevant and interesting topics in mathematics. I would be able to explore how, specifically, we approach research in mathematics. However, I would also be able to experience research of interesting and relevant topics in other disciplines as well. In grade school, there was never a subject that I did not find intriguing or a field in which I did not picture myself working and genuinely enjoying (to the point where choosing what I would study in college had become an issue of deep thought and discussion). However, the more I study mathematics, the more it becomes clear to me that in doing so, you do not have to truly forsake the study of anything. As Dean Schlicter is credited with having said, "Go down deep enough into anything and you will find mathematics." SOAR offers the opportunity for me to explore more about the interrelatedness of all disciplines in experiencing research in various ones.

As an aspiring teacher, SOAR also offers me the ability to, in researching challenging mathematical concepts, be able to better relate with my future students. All students when being introduced to new and unfamiliar concepts experience a state of disequilibrium. It is in such a state, and only in such a state, that psychologist Jean Piaget believed that we learn. Learning new concepts places our minds in a state of confusion which we can only alleviate (and seek to alleviate) by utilizing prior knowledge to draw connections, develop schema (mechanisms to organize and make sense of experience), and eventually begin to understand. As teachers, in particular mathematics teachers, we spend a large portion of our time teaching children what for them is new and daunting material, and what for us, is usually a very familiar concept. It is thereby crucial for me to be willing to put myself in situations that challenge and push my own knowledge so that I can better relate to, connect with, and thus, know how to support my students.

The prospect of working on unsolved mathematical problems is especially important to me. The problems that we work on will be directly applicable to problems that exist in our world and to which an answer would allow for a betterment of various fields, including mathematics. Often when exploring mathematics, young minds inquire about how math relates to anything meaningful in the world, how they will use it when they are older, and how it will be of any benefit to them. We teachers tend to respond with unsatisfying answers such as, "You'll need to go grocery shopping or balance your checkbook when you're older." However, mathematics goes so far beyond the necessary, though fairly monotonous, tasks of daily life. Being able to engage in meaningful research that exemplifies just how expansive and inclusive the value of mathematics is, perhaps, is the most alluring aspect about the SOAR and REU programs. I once had a professor who often mentioned how striking it is that when early mathematics studied sequences of 0s and 1s, they had no dream or even concept of the 20th century programmable computers that we have come to know and love. Without realizing, they made massive contributions not only to the future of mathematics, but to all fields and to the lives of all people, everywhere. They studied mathematics simply for the sake of understanding the world around them, and in that aspiration, we tend always to better it. For me, the research opportunities that SOAR offers is about both contributing to the field of mathematics and inspiring the next generation so that we may continue to have curious minds that are equally passionate about doing the same. It is with experiences like SOAR that I should hope to be able to give my students more inspiring answers to their questions. Answers that reflect that while mathematics is about solving consequential problems and making life more efficient and enjoyable, mathematics is also about looking for answers where there may be none. Mathematics is about solving problems and discovering possibilities that may have no present use. Mathematics is about setting foundations for an unforeseen future, and that is what is captivating and exciting about it.

Expected Outcomes

The expected outcomes of the SOAR project for me is, through research, to challenge and expand my mathematical knowledge. I expect to gain perspective and experience by collaborating with talented minds through the SOAR and REU programs. Furthermore, having had the opportunity to present my research results as well as discuss mathematical research in general to not only math peers and professionals, but also to non-mathematical audiences will prove a vital experience for me as I move to student teach in high schools in the coming semesters. As a teacher, it will be paramount that I be able to coherently and comprehensibly relate mathematical concepts to students and parents. In addition, teaching is most efficient when co-operation and communication take place across and between disciplines to better connect students' understanding of the utility of each and connection with one another, and this is another value of the program, for me. Overall, experiencing the academic environment of research and collaboration will better position me to continue in a life dedicated to the exploration of mathematics, which will enhance my capabilities and passion as a teacher.